Prove the reduction formula $\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\,dx$. **SCORE: _____ / 7 PTS**

NOTE: You must show how to get this formula.
You will receive 0 credit if your "proof" is differentiating both sides of the equation.

u	dv
$\sin^{n-1}x$ 1	$\sin x$ 1
$(n-1)\sin^{n-2}x\cos x$	$-\cos x$

1 1

$$\int \sin^n x\,dx = -\sin^{n-1}x\cos x + (n-1)\int \sin^{n-2}x\cos^2 x\,dx$$

$$= -\sin^{n-1}x\cos x + (n-1)\int \sin^{n-2}x(1-\sin^2 x)\,dx \quad 1\tfrac{1}{2}$$

$$= -\sin^{n-1}x\cos x + (n-1)\int \sin^{n-2}x\,dx - (n-1)\int \sin^n x\,dx$$

1

$$n\int \sin^n x\,dx = -\sin^{n-1}x\cos x + (n-1)\int \sin^{n-2}x\,dx$$

$$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\,dx$$

$\frac{1}{2}$

Evaluate $\int \sqrt{y^2 - 6y + 13}\,dy$. **SCORE: _____ / 7 PTS**

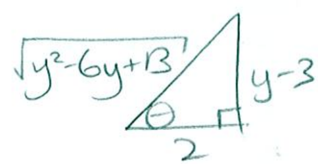
$\tan^2\theta + 1 = \sec^2\theta$

$4\tan^2\theta + 4 = 4\sec^2\theta$

LET $(y-3)^2 = 4\tan^2\theta$

1 $y = 3 + 2\tan\theta \longrightarrow \tan\theta = \frac{y-3}{2}$

$dy = 2\sec^2\theta\,d\theta$



$$= \int \sqrt{(y-3)^2 + 4}\,dy \quad 1$$

$$= \int \sqrt{4\sec^2\theta}\,2\sec^2\theta\,d\theta \quad 1$$

$$= 4\int \sec^3\theta\,d\theta \quad 1$$

$$= 4\left(\tfrac{1}{2}\left(\ln|\sec\theta + \tan\theta| + \sec\theta\tan\theta\right)\right) + C \quad 1$$

$$= 2\left(\ln\left|\frac{\sqrt{y^2-6y+13}}{2} + \frac{y-3}{2}\right| + \frac{\sqrt{y^2-6y+13}}{2}\,\frac{y-3}{2}\right) + C \quad \tfrac{1}{2}\ \tfrac{1}{2}$$

$$= 2\ln\left(\sqrt{y^2-6y+13} + y - 3\right) + \tfrac{1}{2}(y-3)\sqrt{y^2-6y+13} + C \quad \tfrac{1}{2}\ \tfrac{1}{2}$$

Evaluate $\int \sec^4 r \tan^4 r\, dr$. SCORE: ____ / 5 PTS

$u = \tan r$ (1)

$du = \sec^2 r\, dr$

$= \int \sec^2 r \tan^4 r \sec^2 r\, dr$

$= \int (\tan^2 r + 1) \tan^4 r \cdot \sec^2 r\, dr$

$= \int (u^2+1) u^4\, du$ $(1\tfrac{1}{2})$

$= \int (u^6 + u^4)\, du$

$= \frac{1}{7}u^7 + \frac{1}{5}u^5 + C$ (1)

$= \frac{1}{7}\tan^7 r + \frac{1}{5}\tan^5 r + C$ (1) $(\tfrac{1}{2})$

Evaluate $\int \sqrt{m}\,(\ln m)^2\, dm$. SCORE: ____ / 5 PTS

$u = (\ln m)^2 \quad dv = m^{\frac{1}{2}}\, dm$

$du = \frac{2 \ln m}{m}\, dm \quad v = \frac{2}{3} m^{\frac{3}{2}}$

$= \frac{2}{3} m^{\frac{3}{2}} (\ln m)^2 - \frac{4}{3} \int m^{\frac{1}{2}} \ln m\, dm$ (1) (1)

$u = \ln m \quad dv = m^{\frac{1}{2}}\, dm$

$du = \frac{1}{m}\, dm \quad v = \frac{2}{3} m^{\frac{3}{2}}$

$= \frac{2}{3} m^{\frac{3}{2}} (\ln m)^2 - \frac{4}{3}\left(\frac{2}{3} m^{\frac{3}{2}} \ln m - \frac{2}{3} \int m^{\frac{1}{2}}\, dm\right)$ (1) $(\tfrac{1}{2})$

$= \frac{2}{3} m^{\frac{3}{2}} (\ln m)^2 - \frac{8}{9} m^{\frac{3}{2}} \ln m + \frac{16}{27} m^{\frac{3}{2}} + C$ $(\tfrac{1}{2})$ $(\tfrac{1}{2})$ $(\tfrac{1}{2})$

$= \frac{2}{27} m^{\frac{3}{2}} (9(\ln m)^2 - 12 \ln m + 8) + C$

SEE TABLE METHOD SOLUTION AT END OF FILE FOR ALTERNATE METHOD

Using the integral based definition of $LN(x)$, prove that $LN(\frac{x}{y}) = LN(x) - LN(y)$ using substitution. SCORE: ____ / 6 PTS
(Substitution was the technique used in the proof in lecture.)

$LN\left(\frac{x}{y}\right) = \int_1^{\frac{x}{y}} \frac{1}{t}\, dt = \int_1^x \frac{1}{t}\, dt + \int_x^{\frac{x}{y}} \frac{1}{t}\, dt$ (1) (1)

$= \int_1^x \frac{1}{t}\, dt - \int_{\frac{x}{y}}^x \frac{1}{t}\, dt$

LET $u = \frac{y}{x} t$ (1)

$t = \frac{xu}{y}$

$t = x \Rightarrow u = y$

$t = \frac{x}{y} \Rightarrow u = 1$

$\frac{du}{dt} = \frac{y}{x}$

$dt = \frac{x}{y}\, du$

$= \int_1^x \frac{1}{t}\, dt - \int_1^y \frac{y}{xu} \cdot \frac{x}{y}\, du$ (2)

$= \int_1^x \frac{1}{t}\, dt - \int_1^y \frac{1}{u}\, du$

$= LN(x) - LN(y)$ (1)

WATCH OUT FOR ALL LIMITS OF INTEGRATION

☆ TABLE METHOD

$$\int \sqrt{m}\,(\ln m)^2\,dm = \underbrace{\frac{2}{3}m^{\frac{3}{2}}(\ln m)^2}_{\frac{1}{2}} - \underbrace{\frac{8}{9}m^{\frac{3}{2}}(\ln m)^2}_{\frac{1}{2}} + \underbrace{\frac{16}{27}m^{\frac{3}{2}}}_{\frac{1}{2}} + \underbrace{C}_{\frac{1}{2}}$$

u		dv
$(\ln m)^2$	$+$	$m^{\frac{1}{2}}$
$\frac{2\ln m}{m}$		$\frac{2}{3}m^{\frac{3}{2}}$ ①
$*m$		$*\frac{1}{m}$
$2\ln m$		$\frac{2}{3}m^{\frac{1}{2}}$
$\frac{2}{m}$	$-$	$\frac{4}{9}m^{\frac{3}{2}}$ ①
$*m$		$*\frac{1}{m}$
2		$\frac{4}{9}m^{\frac{1}{2}}$
0	$+$	$\frac{8}{27}m^{\frac{3}{2}}$ ①